Unified relativistic description of πNN and $\gamma\pi NN$

A. N. Kvinikhidze* and B. Blankleider
Department of Physics, The Flinders University of South Australia, Bedford Park, SA 5042, Australia

Abstract

We present a unified description of the relativistic πNN and $\gamma\pi NN$ systems where the strong interactions are described non-perturbatively by four-dimensional integral equations. A feature of our approach is that the photon is coupled in all possible ways to the strong interaction contributions. Thus the hadronic processes $NN\rightarrow NN$, $NN\rightarrow \pi d$, $\pi d\rightarrow \pi d$, etc. and corresponding electromagnetic processes $NN\rightarrow \gamma NN$, $\gamma d\rightarrow NN$, $\gamma d\rightarrow \pi d$, $ed\rightarrow ed$, $ed\rightarrow e'\pi d$, etc., are described simultaneously within the one model of strong interactions. Our formulation obeys two and three-body unitarity, and as photons are coupled everywhere in the strong interaction model, gauge invariance is implemented in the way prescribed by quantum field theory. Our formulation is also free from the overcounting and undercounting problems plaguing four-dimensional descriptions of πNN-like systems. The unified description is achieved through the use of the recently introduced gauging of equations method.

I. INTRODUCTION

Recently we have introduced a method for incorporating an external electromagnetic field into any model of hadrons whose strong interactions are described through the solution of integral equations [1,2]. The method involves the gauging of the integral equations themselves, and results in electromagnetic amplitudes where an external photon is coupled to every part of every strong interaction diagram in the model. Current conservation is therefore implemented in the theoretically correct fashion, i.e. as prescribed by quantum field theory. Initially we applied our gauging procedure to the relativistic three-nucleon problem whose strong interactions are described by standard four-dimensional three-body integral equations [1,2]. More recently we used the same method to gauge the three-dimensional spectator equation for a system of three-nucleons [3,4].

Here we apply our gauging procedure to the more complicated case of the relativistic πNN system whose four-dimensional integral equations have only recently been derived [6,7].[1] These equations obey both 2-body (NN) and three-body (πNN) unitarity and simultaneously describe all the strong interaction processes of the πNN system, including the reactions $NN\rightarrow NN$,

---

*On leave from The Mathematical Institute of Georgian Academy of Sciences, Tbilisi, Georgia.

[1] A summary of the present work was previously reported in a conference proceeding [5]

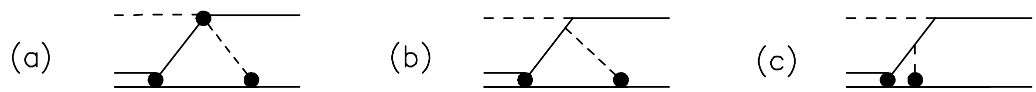

FIG. 1. Example of overcounting in $NN \to \pi d$. (a) The $NN \to \pi d$ Feynman diagram where dark circles represent all possible contributions. (b) One of the contributions included in (a). (c) Another way of drawing diagram (b) showing that this term corresponds to an overdressing of the deuteron vertex.

$NN \to \pi d$, $NN \to \pi NN$, $\pi d \to \pi d$ and $\pi d \to \pi NN$. After gauging these πNN equations we obtain gauge invariant expressions for all possible electromagnetic processes of the πNN system, e.g. pion photoproduction $\gamma d \to \pi d$ and $\gamma d \to \pi NN$, pion electroproduction $ed \to e'\pi d$ and $ed \to e'\pi NN$, and because pion absorption is taken into account explicitly, we also obtain gauge invariant expressions for processes like deuteron photodisintegration $\gamma d \to NN$ and Bremsstrahlung $NN \to \gamma NN$ that are valid even at energies above pion production threshold. Included in the electromagnetic processes described by our model is the especially interesting case of elastic electron-deuteron scattering. Here the deuteron is described within our model as a bound state of the πNN system; thus, after gauging, our model provides a rich description of the electromagnetic form factors of the deuteron with all possible meson exchange currents taken into account. The four-dimensional integral equations for the πNN system are a good deal more complicated than the corresponding equations describing a strictly three-body system like that of three nucleons. Firstly the πNN equations take into account pion absorption so there is coupling to the two-body NN channel. A consequence of this in the four-dimensional approach is that certain three-body forces must be retained both to avoid the undercounting of diagrams and to ensure that the equations satisfy two- and three-body unitarity [6]. Also, unlike in the NNN case, not all three-body reducible diagrams of the πNN system have unique three-body cuts. This leads to an overcounting of diagrams problem in the formulation of four-dimensional πNN equations that has only recently been solved [6]. One example of the type of overcounting encountered is given in Fig. 1. The newly derived πNN equations have the feature that their kernels contain explicit subtraction terms which eliminate all such overcounting.

The presence of pion absorption and the overcounting problem in the πNN system would normally make the task of formulating a description of the $\gamma\pi NN$ system an especially difficult one. To put this difficulty into perspective, we recall our experience with gauging the four-dimensional three-nucleon system [1,2]. In this case the strong interaction integral equations have exactly the same form as the three-body equations of quantum mechanics (QM) (except of course that they are four-dimensional rather than three-dimensional). Then after applying our gauging procedure we obtained an electromagnetic current that is again just of the same form as the three-nucleon current in QM; namely, a simple sum of one and two-body currents. However, in contrast to the QM case, the two-body current obtained in the four-dimensional formulation is modified by a subtraction term whose presence is necessary to stop the overcounting of diagrams. Thus the gauging of equations method led us to a simple prescription for the electromagnetic current of three nucleons. The situation for πNN is quite different as the strong interaction equations for this system cannot be cast into a QM form. Thus there is no corresponding simple prescription for the πNN electromagnetic current and the gauging procedure itself becomes effectively the only way to specify this current. On the other hand, our gauging procedure is extremely simple, and by gauging the πNN equations with subtraction terms included, one easily constructs equations for the $\gamma\pi NN$ system without encountering any

further difficulties and with all overcounting problems being taken care of automatically. In this way we obtain a unified description of the πNN and $\gamma\pi NN$ systems.

II. FOUR-DIMENSIONAL πNN EQUATIONS

The first attempts to formulate few-body equations using relativistic quantum field theory were made already in the early 1960's [8–10]. Both such general formulations and ones more specific to the πNN system have been pursued until the present time [11–13]. Yet all these attempts have had theoretical inconsistencies, including the undercounting and overcounting problems discussed above. We have recently overcome these problems and derived new consistent πNN equations by using a method where in cases like that of Figs. 1(b) and (c) where the rightmost πNN cut is not unique, one of the rightmost πNN vertices is "pulled out" further to the right, in this way defining a unique πNN cut [6]. The same πNN equations were later derived in Ref. [7] where a method based on Taylor's classification of diagrams [8,14] was used. In this section we would simply like to restate these equations but in a form that is particularly convenient for gauging. In this section we would simply like to restate these equations but in a form that is particularly convenient for gauging.

A. Distinguishable nucleon case

Initially we would like to consider the case where the two nucleons are treated as distinguishable particles. Not only does this avoid taking into account the symmetry factors and related complications due to the identity of the two nucleons, but it is also of practical interest in itself, as for example in modelling the πnp system without using isospin symmetry.

We follow the usual convention and refer to the two nucleons as particles 1 and 2, and the pion as particle 3. We also use $\lambda = 1$ or 2 to label the channel where nucleon λ and the pion form a two-particle subsystem with the other nucleon being a spectator, and $\lambda = 3$ to label the channel where the two nucleons form a subsystem with the pion being a spectator. Although we are concerned only with those physical processes having at most one pion in either initial or final state, we do allow multiple-pion intermediate states. These intermediate state pions may be taken to be either distinguishable or indistinguishable without affecting the formulation below in any essential way.

Although the derivation of the πNN equations given in Ref. [6] neglected all connected diagrams that are both NN- and πNN-irreducible in the processes $NN \to NN$ and $NN \leftrightarrow \pi NN$, in our approach these are easily included and do not complicate the original πNN equations in any essential way; for this reason we shall retain all such diagrams here. On the other hand we follow Ref. [6] and keep only those NN- and πNN-irreducible connected diagrams in the process $\pi NN \to \pi NN$ (the three-body forces) that are necessary to avoid undercounting. The formulation retaining *all* three-body forces will be given elsewhere.

It is easy to rearrange the four-dimensional πNN equations of Ref. [6] into a convenient form similar to the one used by Afnan and Blankleider [15] in a three-dimensional formulation of the πNN system. For the distinguishable nucleon case we obtain

$$\mathcal{T}^d = \mathcal{V}^d + \mathcal{V}^d \mathcal{G}_t^d \mathcal{T}^d \tag{1}$$

where $\mathcal{T}^d$, $\mathcal{V}^d$, and $\mathcal{G}_t^d$ are 4×4 matrices given by

$$\mathcal{T}^d = \begin{pmatrix} T^d_{NN} & \bar{T}^d_N \\ T^d_N & T^d \end{pmatrix}; \quad \mathcal{V}^d = \begin{pmatrix} V^d_{NN} & \bar{\mathcal{F}}^d \\ \mathcal{F}^d & G_0^{-1}\mathcal{I} \end{pmatrix}; \quad \mathcal{G}^d_t = \begin{pmatrix} D_0 & 0 \\ 0 & G_0 w^0 G_0 \end{pmatrix}. \tag{2}$$

Note that we use a notation where the superscript d denotes the distinguishable nucleon case on those symbols that will be used later without the superscript for indistinguishable nucleons. Eq. (1) is a symbolic equation representing a Bethe-Salpeter integral equation to be solved for $\mathcal{T}^d$. To clarify its meaning we give the explicit numerical form of the equation for NN scattering at the end of this subsection. $\mathcal{T}^d$ consists of transition amplitudes T^d_{NN}, $T_{\lambda N}$, $T_{N\mu}$, and $T_{\lambda\mu}$ (λ and μ are spectator-subsystem channel labels) the last three being elements of the matrices T^d_N, $\bar{T}^d_N$, and T^d, respectively. The physical amplitudes for $NN \to NN$, $NN \to \pi d$, $\pi d \to NN$, and $\pi d \to \pi d$ are then given by

$$X^d_{NN} = T^d_{NN} \quad ; \quad X^d_{dN} = \bar{\psi}_d T_{3N} \quad ; \quad X^d_{Nd} = T_{N3}\psi_d \quad ; \quad X^d_{dd} = \bar{\psi}_d T_{33}\psi_d, \tag{3}$$

respectively, where ψ_d is the deuteron wave function in the presence of a spectator pion.

The elements making up the kernel $\mathcal{V}^d$, specified in Eq. (2), consist of the quantities V^d_{NN}, $\mathcal{F}^d_\lambda$, $\bar{\mathcal{F}}^d_\lambda$, and $\mathcal{I}$ defined as follows:

$$\begin{aligned} V^d_{NN} = {} & V^{\mathrm{OPE}d}_{NN} + V^{(1)\,d}_{NN} + \bar{F}^d_c G_0(F_1 + F_2) + (\bar{F}_1 + \bar{F}_2)G_0 F^d_c \\ & + \bar{B}^d G_0 B^d + \bar{F}^d_c G_0 F^d_c - \Delta \end{aligned} \tag{4}$$

where $V^{\mathrm{OPE}d}_{NN}$ is the nucleon-nucleon one pion exchange potential, $V^{(1)\,d}_{NN}$ is the πNN-irreducible part of the NN potential, $F_i = f_i d_j^{-1}$ where $i, j = 1, 2$ with $i \neq j$, f_i being the $N_i \to \pi N_i$ vertex function and d_j the Feynman propagator of nucleon j, F^d_c is the simultaneously NN- and πNN-irreducible connected amplitude for $NN \to \pi NN$, $B^d = B + PBP$ where $B = V^{\mathrm{OPE}d}_{NN} d_2 f_2$ and P is the nucleon exchange operator, and Δ is a subtraction term that eliminates overcounting. Diagrams illustrating $V^{\mathrm{OPE}d}_{NN}$, $V^{(1)\,d}_{NN}$, F_2, B and F^d_c are given in Fig. 2. $\mathcal{F}^d$ is a 3×1 matrix whose λ'th row element is given by

$$\mathcal{F}^d_\lambda = \sum_{i=1}^{2} \bar{\delta}_{\lambda i} F_i + F^d_c - B^d \tag{5}$$

where $\bar{\delta}_{\lambda i} = 1 - \delta_{\lambda i}$. Note that here B^d plays the role of a subtraction term. $\bar{\mathcal{F}}^d$ is the 1×3 matrix that is the time reversed version of $\mathcal{F}^d$ (similarly for other "barred" quantities), G_0 is the πNN propagator, and $\mathcal{I}$ is the matrix whose (λ, μ)'th element is $\bar{\delta}_{\lambda,\mu}$. Finally the propagator term $\mathcal{G}^d_t$ is a diagonal matrix consisting of the NN propagator D_0, and the 3×3 diagonal matrix w^0 whose diagonal elements are $t_1 d_2^{-1}$, $t_2 d_1^{-1}$, and $t^d_3 d_3^{-1}$, with t_λ being the two-body t matrix

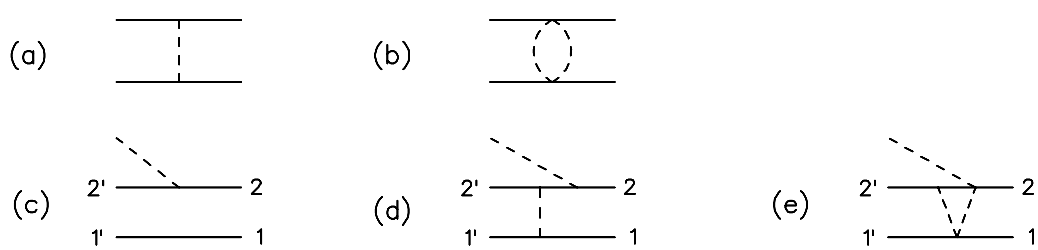

FIG. 2. Illustration of quantities making up V^d_{NN}. (a) $V^{\mathrm{OPE}d}_{NN}$, (b) a contribution to $V^{(1)\,d}_{NN}$, (c) $F_2 = f_2 d_1^{-1}$, (d) B, and (e) a contribution to F^d_c.

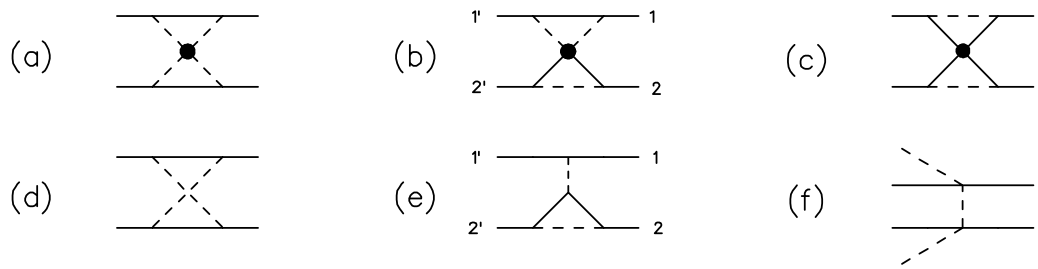

FIG. 3. Terms making up the subtraction term Δ. (a) $W_{\pi\pi}$, (b) $W_{\pi N}$, (c) W_{NN}, (d) X, (e) Y, and (f) a contribution to $F_{2\pi}$. The dark circles represent the following two-body amplitudes: (a) full $\pi\pi$ t-matrix, (b) one-nucleon irreducible πN t-matrix, and (c) full NN t-matrix minus the NN one-pion-exchange potential.

in channel λ (for $\lambda = 1$ or 2, t_λ is defined to be the πN t matrix with the nucleon pole term removed). The subtraction term Δ is defined with the help of Fig. 3 as follows:

$$\begin{aligned}\Delta = {} & W_{\pi\pi} + W^d_{\pi N} + W_{NN} + X + Y^d \\ & + \bar{B}^d G_0 F^d_c + \bar{F}^d_c G_0 B^d + \bar{F}_{2\pi} G^{2\pi}_0 f_1 f_2\Big|_{\rm ND} + \bar{f}_1 \bar{f}_2 G^{2\pi}_0 F_{2\pi}\Big|_{\rm ND}\end{aligned} \tag{6}$$

where $F_{2\pi}$ is the simultaneously NN- and πNN-irreducible connected amplitude for $NN \rightarrow \pi\pi NN$, $G^{2\pi}_0$ is the $\pi\pi NN$ propagator, $W^d_{\pi N} = W_{\pi N} + PW_{\pi N}P$ and $Y^d = Y + PYP$. The expression $\bar{f}_1\bar{f}_2 G^{2\pi}_0 F_{2\pi}$ in Eq. (6) is a subtraction term for the overcounted contributions in $(\bar{F}_1 + \bar{F}_2) G_0 F^d_c$ in Eq. (4). It consists of two types of contributions, one with the two intermediate state pions crossing, and one with them not crossing, as illustrated in Fig. 4. As our equations are derived with the πNN vertex being dressed from the beginning [6], terms that contribute to a further dressing of the πNN vertex, like that of Fig. 4(b), need to be suppressed. This suppression is indicated in Eq. (6) by the "no dressing" symbol $|_{\rm ND}$. Similar comments of course apply to the term $\bar{F}_{2\pi} G^{2\pi}_0 f_1 f_2$ in Eq. (6).

To illustrate the numerical form of our equations we consider the case of the NN scattering amplitude T^d_{NN} as given by Eq. (1) and Eq. (2)

$$T^d_{NN} = V^d_{NN} + V^d_{NN} D_0 T^d_{NN} + \sum_{\lambda=1}^{3} \bar{\mathcal{F}}^d_\lambda G_0 w^0_{\lambda\lambda} G_0 T^d_{\lambda N}. \tag{7}$$

For the purposes of the illustration we may simplify this equation by neglecting F^d_c (but not B^d as it is a subtraction term). In this case we obtain the following numerical integral equation for T^d_{NN}:

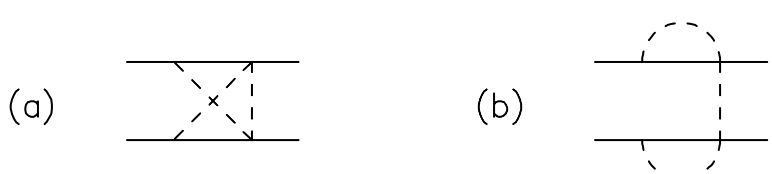

FIG. 4. Example of diagrams contributing to $\bar{f}_1\bar{f}_2 G^{2\pi}_0 F_{2\pi}$ in Eq. (6). (a) Contribution with crossed pions, (b) contribution with pions not crossing.

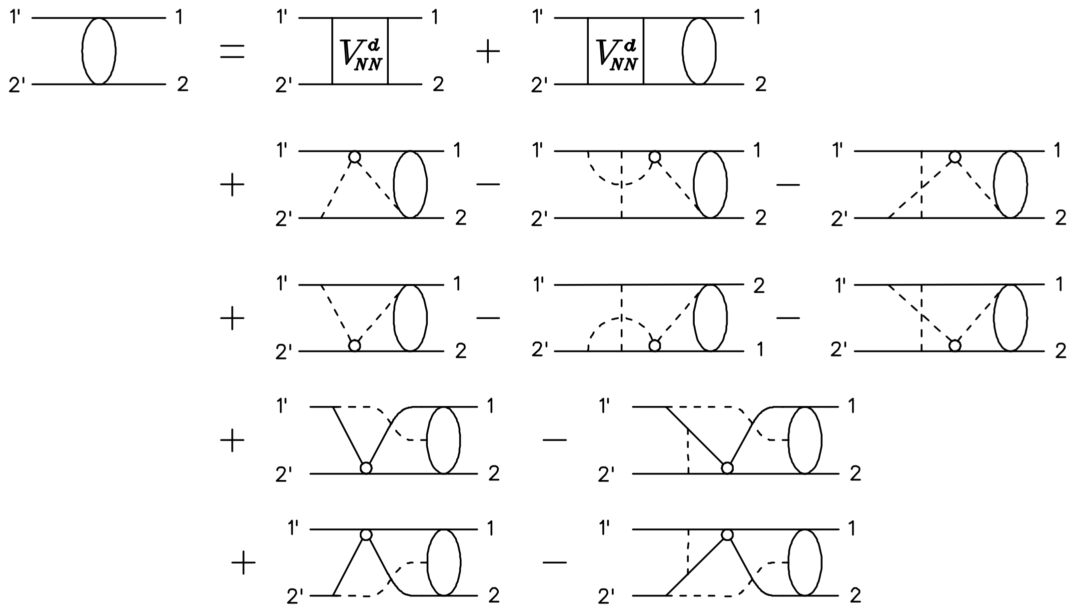

FIG. 5. Graphical representation of the integral equation, Eq. (8), for the NN scattering amplitude. The top line involves the amplitude T^d_{NN}, the second line involves T_{1N}, the third line involves T_{2N} and the last two lines involve T_{3N}.

$$
\begin{aligned}
T^d_{NN}(p'_1p'_2,p_1p_2) &= V^d_{NN}(p'_1p'_2,p_1p_2) + \int \frac{d^4k_1}{(2\pi)^4} V^d_{NN}(p'_1p'_2,k_1k_2)D_0(k_1k_2)T^d_{NN}(k_1k_2,p_1p_2) \\
&+ \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\Big\{ \bar{f}_2(p'_2,k_2k'_3)d_\pi(k'_3)t_1(p'_1k'_3,k_1k_3) \\
&\quad - \int \frac{d^4k'_1}{(2\pi)^4}\Big[\bar{f}_1(p'_1,k''_1k'_3)D_{\pi N}(k''_1k'_3)V^{\rm OPE}_{NN}(k''_1p'_2,k'_1k_2) \\
&\quad + \bar{f}_2(p'_2,k'_2k'_3)D_{\pi N}(k'_2k'_3)V^{\rm OPE}_{NN}(p'_1k'_2,k'_1k_2)\Big] \\
&\quad \times d_1(k'_1)t_1(k'_1k'_3,k_1k_3)\Big\} G_0(k_1k_2k_3)T_{1N}(k_1k_2k_3,p_1p_2) + \begin{pmatrix} p'_1 \leftrightarrow p'_2 \\ p_1 \leftrightarrow p_2 \end{pmatrix} \\
&+ \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{f}_1(p'_1,k'_1k_3)d_1(k'_1)\Big[t_3(k'_1p'_2,k_1k_2) \\
&\quad - \int \frac{d^4k''_1}{(2\pi)^4}V^{\rm OPE}_{NN}(k'_1p'_2,k''_1k'_2)D_0(k''_1k'_2)t_3(k''_1k'_2,k_1k_2)\Big] \\
&\quad \times G_0(k_1k_2k_3)T_{3N}(k_1k_2k_3,p_1p_2) + \begin{pmatrix} p'_1 \leftrightarrow p'_2 \\ p_1 \leftrightarrow p_2 \end{pmatrix} \qquad (8)
\end{aligned}
$$

where to save on notation we have suppressed spin and isospin labels and used the symbol $\begin{pmatrix} p'_1 \leftrightarrow p'_2 \\ p_1 \leftrightarrow p_2 \end{pmatrix}$ to indicate a term derived from the preceding contribution by exchanging the momenta, spin, and isospin labels of the external nucleons. In writing Eq. (8) we have used the fact that numerically $\bar{f}_1(p',pk) = \bar{f}_2(p',pk)$, $t_1(p'k',pk) = t_2(p'k',pk)$, and $T_{1N}(k_2k_1k_3,p_2p_1) = T_{2N}(k_1k_2k_3,p_1p_2)$. In Eq. (8) $D_{\pi N}(k_1k_3) = d_1(k_1)d_\pi(k_3)$ is the πN propagator, and all unspecified momenta are determined by momentum conservation, e.g. in all the above terms $k_3 = p_1 + p_2 - k_1 - k_2$. Eq. (8) is illustrated in Fig. 5. In reference to this figure

we note that despite appearances, all the subtraction terms (terms entering with a minus sign) end on the left in the same way, either with $\bar{B} = \bar{f}_2 d_2 V_{NN}^{\rm OPE}$ or with $P\bar{B}P = \bar{f}_1 d_1 V_{NN}^{\rm OPE}$. Also, the last two lines clearly involve the contribution $t_3 - V_{NN}^{\rm OPE} D_0 t_3$ which may appear incorrect as the term subtracted involves an overcounted NN interaction. Yet just such an overcounted contribution needs to be subtracted to stop overcounting in the overall equations. In this respect it is interesting to note that in the one pion exchange ladder approximation to t_3, the combination $t_3 - V_{NN}^{\rm OPE} D_0 t_3$ reduces down simply to $V_{NN}^{\rm OPE}$.

B. Indistinguishable nucleon case

In approaches based on second quantisation in quantum mechanics it is usual to obtain the scattering equations for identical particles by explicitly symmetrizing the equations of the distinguishable particle case. This procedure is not justified in the framework of relativistic quantum field theory. Nevertheless, as we have already derived the πNN equations taking into account identical particle symmetry right from the beginning [6], we can formally deduce how the above distinguishable nucleon πNN equations need to be modified in order to get the indistinguishable nucleon case. With this in mind, we introduce identical nucleon transition amplitudes defined in terms of distinguishable nucleon transition amplitudes as

$$\begin{array}{lll} T_{NN} = T^d_{NN} A & T_{\Delta N} = T_{1N} A & T_{dN} = T_{3N} A \\ T_{N\Delta} = \bar{T}_{N1} - \bar{T}_{N2} P & T_{Nd} = \bar{T}_{N3} A & T_{\Delta\Delta} = T_{11} - T_{12} P \\ T_{d\Delta} = T_{31} - T_{32} P & T_{\Delta d} = T_{13} A & T_{dd} = T_{33} A \end{array} \tag{9}$$

where $A = 1 - P$ is the antisymmetrizing operator. As $\bar{T}_{N1} = P\bar{T}_{N2}P$ and $\bar{T}_{N3} = P\bar{T}_{N3}P$, we can alternatively write $T_{N\Delta} = A\bar{T}_{N1}$, $T_{Nd} = A\bar{T}_{N3}$, and $T_{d\Delta} = A\bar{T}_{31}$. Thus in the transition to the indistinguishable particle case, the original 16 transition amplitudes for distinguishable particles have been reduced to 9 antisymmetric transition amplitudes. By taking residues of the πNN Green function for identical nucleons at two-body subsystem poles, one obtains the following expressions for the physical amplitudes:

$$X_{NN} = T_{NN} \ ; \quad X_{dN} = \bar{\psi}_d T_{dN} \ ; \quad X_{Nd} = T_{Nd}\psi_d \ ; \quad X_{dd} = \bar{\psi}_d T_{dd} \psi_d. \tag{10}$$

Using Eqs. (9) in Eq. (1) it is easy to show that one again obtains a Bethe-Salpeter equation

$$\mathcal{T} = \mathcal{V} + \mathcal{V}\mathcal{G}_t\mathcal{T} \tag{11}$$

but where now $\mathcal{T}$, $\mathcal{V}$, and $\mathcal{G}_t$ are 3×3 matrices given by

$$\mathcal{T} = \begin{pmatrix} T_{NN} & T_{N\Delta} & T_{Nd} \\ T_{\Delta N} & T_{\Delta\Delta} & T_{\Delta d} \\ T_{dN} & T_{d\Delta} & T_{dd} \end{pmatrix}; \qquad \mathcal{V} = \begin{pmatrix} V^d_{NN} A & A\bar{\mathcal{F}}^d_1 & \bar{\mathcal{F}}^d_3 A \\ \mathcal{F}^d_1 A & -G_0^{-1} P & G_0^{-1} A \\ \mathcal{F}^d_3 A & G_0^{-1} A & 0 \end{pmatrix}; \tag{12}$$

$$\mathcal{G}_t = \begin{pmatrix} \frac{1}{2} D_0 & 0 & 0 \\ 0 & G_0 t_1 d_2^{-1} G_0 & 0 \\ 0 & 0 & \frac{1}{4} G_0 t_3 d_3^{-1} G_0 \end{pmatrix} \tag{13}$$

where $t_3 = t_3^d A$ is the t matrix for two identical nucleons in the presence of a spectator pion.

From Eq. (4) it follows that

$$
\begin{aligned}
V_{NN}^d A = {} & V_{NN}^{\mathrm{OPE}} + V_{NN}^{(1)} + \bar{F}_c G_0 F_1^R + \bar{F}_1^L G_0 F_c \\
& + \bar{F}_{2\pi}^L G_0^{2\pi} f_1 f_2 + \bar{f}_1 \bar{f}_2 G_0^{2\pi} F_{2\pi}^R - W_{\pi\pi}^R - W_{\pi N}^{LR} - W_{NN}^R - X^R - Y^{LR} \\
& + \frac{1}{2}(\bar{B}^{LR} - \bar{F}_c) G_0 (B^{LR} - F_c)
\end{aligned} \tag{14}
$$

where $V_{NN}^{\mathrm{OPE}} = V_{NN}^{\mathrm{OPE}d} A$, $V_{NN}^{(1)} = V_{NN}^{(1)d} A$, $F_c = F_c^d A$, $\bar{F}_c = \bar{F}_c^d A$, and where we have used a superscript L to indicate that A is acting on the left, and a superscript R to indicate that A is acting on the right. Note that $W_{\pi\pi}^R = W_{\pi\pi}^L$, $W_{NN}^R = W_{NN}^L$, $X^R = X^L$, and $F_{2\pi}^R = F_{2\pi}^L$. Here we have also used the simple result

$$
(\bar{B}^d - \bar{F}_c^d) G_0 (B^d - F_c^d) A = \frac{1}{2}(\bar{B}^{LR} - \bar{F}_c) G_0 (B^{LR} - F_c). \tag{15}
$$

The other terms in the kernel $\mathcal{V}$ of Eq. (12) are

$$
\mathcal{F}_1^d = F_2 - B^d + F_c^d; \qquad \bar{\mathcal{F}}_1^d = \bar{F}_2 - \bar{B}^d + \bar{F}_c^d, \tag{16}
$$

$$
\mathcal{F}_3^d = F_1 + F_2 - B^d + F_c^d; \qquad \bar{\mathcal{F}}_3^d = \bar{F}_1 + \bar{F}_2 - \bar{B}^d + \bar{F}_c^d. \tag{17}
$$

The πNN equations as given by Eqs. (11), (12) and (13) have a form where seven of the eight non-zero elements making up the kernel $\mathcal{V}$ contain the antisymmetrization operator A. One element (t_3) in $\mathcal{G}_t$ also contains A. The question arises if one can remove the operators A from these equations by analogy with the familiar example of the one-channel Bethe-Salpeter equation for two identical nucleons:

$$
T = VA(1 + \frac{1}{2} GT). \tag{18}
$$

Here the NN potential V is multiplied on the right by A, but as V has the property $VP = PV$ and therefore $VA = AV$, it is easy to see that we can instead solve the equation

$$
T' = V(1 + GT') \tag{19}
$$

where A has been removed from the equation and where the NN t matrix T is obtained by antisymmetrizing the solution: $T = T'A$.

Unfortunately it is not possible to follow a similar procedure to completely remove the operator A from Eqs. (11), (12) and (13). The reason is that $\mathcal{V}P \neq P\mathcal{V}$ for the kernel $\mathcal{V}$ of Eq. (12) (in particular $\mathcal{F}_1^d P \neq P \mathcal{F}_1^d$). In this respect we note that the πNN equations for identical particles given in Ref. [7] are not equivalent to ours as they do not involve the operator A. Although A cannot be removed completely from the identical particle πNN equations, one can reduce the number of places where A appears. For example, it is easy to show that Eqs. (11), (12) and (13) are equivalent to the equations

$$
\mathcal{T}' = \mathcal{V}' + \mathcal{V}' \mathcal{G}_t' \mathcal{T}'; \tag{20}
$$

$$
\mathcal{V}' = \begin{pmatrix} V_{NN}^d & \bar{\mathcal{F}}_1^d & \bar{\mathcal{F}}_3^d \\ \mathcal{F}_1^d A & -G_0^{-1} P & G_0^{-1} A \\ \mathcal{F}_3^d & G_0^{-1} & 0 \end{pmatrix}; \quad \mathcal{G}_t' = \begin{pmatrix} D_0 & 0 & 0 \\ 0 & G_0 t_1 d_2^{-1} G_0 & 0 \\ 0 & 0 & G_0 t_3^d d_3^{-1} G_0 \end{pmatrix} \tag{21}
$$

where

$$\mathcal{T} = \begin{pmatrix} A & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & A \end{pmatrix} T' \begin{pmatrix} \frac{A}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{A}{2} \end{pmatrix} \tag{22}$$

and all but two operators A have been removed form the integral equations. There are further forms of the πNN equations with operators A appearing at various other places in the equations, yet there are always at least two A operators present.

III. πNN ELECTROMAGNETIC TRANSITION CURRENTS

A. Gauging the πNN equations

In this section we shall derive expressions for the various electromagnetic transition currents of the πNN system. To do this we utilise the recently introduced gauging of equations method [1,2]. As the gauging procedure is identical for the distinguishable and indistinguishable particle cases, we restrict our attention to the πNN system where the nucleons are treated as indistinguishable particles.

As discussed in the previous section, the strong interaction πNN equations can be written in a number of equivalent forms. Choosing the form given by Eqs. (11), (12) and (13), direct gauging of Eq. (11) gives

$$\mathcal{T}^\mu = \mathcal{V}^\mu + \mathcal{V}^\mu \mathcal{G}_t \mathcal{T} + \mathcal{V}\mathcal{G}_t^\mu \mathcal{T} + \mathcal{V}\mathcal{G}_t \mathcal{T}^\mu \tag{23}$$

which can easily be solved for $\mathcal{T}^\mu$ giving

$$\mathcal{T}^\mu = (1 + \mathcal{T}\mathcal{G}_t)\mathcal{V}^\mu(1 + \mathcal{G}_t\mathcal{T}) + \mathcal{T}\mathcal{G}_t^\mu\mathcal{T}. \tag{24}$$

$\mathcal{T}^\mu$ is a matrix of gauged transition amplitudes T^μ_{NN}, $T^\mu_{N\Delta}$, T^μ_{Nd}, etc. To obtain the physical electromagnetic transition currents of the πNN system where photons are attached everywhere it is not sufficient to just gauge the physical πNN amplitudes of Eq. (10). Although this would indeed attach photons everywhere inside the strong interaction diagrams, it would miss those contributions to the physical electromagnetic transition currents where the photons are attached to the external (initial and final state) pions and nucleons. In order to also include these external leg contributions it is useful to attach the corresponding propagators to the X-amplitudes of Eq. (10):

$$\tilde{X}_{NN} = D_0 X_{NN} D_0 \qquad \tilde{X}_{dN} = d_\pi X_{dN} D_0 \tag{25}$$
$$\tilde{X}_{Nd} = D_0 X_{Nd}\, d_\pi \qquad \tilde{X}_{dd} = d_\pi X_{dd}\, d_\pi. \tag{26}$$

The physical electromagnetic transition currents are then obtained by gauging Eqs. (25) and (26) and "chopping off" external legs:

$$j^\mu_{NN} = D_0^{-1} \tilde{X}^\mu_{NN} D_0^{-1} \qquad j^\mu_{dN} = d_\pi^{-1} \tilde{X}^\mu_{dN} D_0^{-1} \tag{27}$$
$$j^\mu_{Nd} = D_0^{-1} \tilde{X}^\mu_{Nd}\, d_\pi^{-1} \qquad j^\mu_{dd} = d_\pi^{-1} \tilde{X}^\mu_{dd}\, d_\pi^{-1}. \tag{28}$$

Using Eqs. (10) we obtain that

$$j^\mu_{NN} = D_0^{-1} D_0^\mu T_{NN} + T_{NN} D_0^\mu D_0^{-1} + T^\mu_{NN} \tag{29}$$

$$\begin{aligned} j^\mu_{dN} &= \bar\phi^\mu_d D_0 T_{dN} + d_\pi^{-1} \bar\phi_d G_0^\mu T_{dN} + \bar\phi_d D_0 T^\mu_{dN} \\ &+ \bar\phi_d D_0 T_{dN} D_0^\mu D_0^{-1} \end{aligned} \tag{30}$$

$$\begin{aligned} j^\mu_{Nd} &= T_{Nd} D_0 \phi^\mu_d + T_{Nd} G_0^\mu \phi_d d_\pi^{-1} + T^\mu_{Nd} D_0 \phi_d \\ &+ D_0^{-1} D_0^\mu T_{Nd} D_0 \phi_d \end{aligned} \tag{31}$$

$$\begin{aligned} j^\mu_{dd} &= \bar\phi^\mu_d D_0 T_{dd} D_0 \phi_d + d_\pi^{-1} \bar\phi_d D_0^\mu T_{dd} D_0 \phi_d \\ &+ \bar\phi_d D_0 T^\mu_{dd} D_0 \phi_d + \bar\phi_d D_0 T_{dd} D_0^\mu \phi_d d_\pi^{-1} \\ &+ \bar\phi_d D_0 T_{dd} D_0 \phi^\mu_d \end{aligned} \tag{32}$$

where ϕ_d is the deuteron bound state vertex function defined by the relation $\psi_d = d_1 d_2 \phi_d$. The above equations express the physical electromagnetic transition currents $j^\mu_{\alpha\beta}$ ($\alpha, \beta = N$ or d) in terms of the half-on-shell πNN transition amplitudes $T_{\alpha\beta}$ and the gauged quantities $V^\mu_{\alpha\beta}$, $(\mathcal{G}^\mu_t)_{\alpha\beta}$, and ϕ^μ_d. Note that ϕ^μ_d consists of contributions where the photon is attached everywhere inside the deuteron bound state, and is determined by gauging the two-nucleon bound state equation for ϕ_d [1,2].

That the above equations are gauge invariant is evident from the fact that we have formally attached the photon to all possible places in the strong interaction model. The gauge invariance of our equations also follows from a strict mathematical proof; however, as this proof is essentially identical to the one given for the NNN system [2], we shall not repeat it here.

B. Alternative form of the πNN equations

Although the preceding discussion solves the problem of gauging the πNN system, the expression obtained for the gauged transition amplitudes, Eq. (24), may not be the most convenient for numerical calculations. The disadvantage of Eq. (24) is that it utilises a Green function $\mathcal{G}_t$ which contains two-body t matrices, while such t matrices are already implicitly present in the adjoining amplitudes of $\mathcal{T}$. This makes the calculation of $\mathcal{T}^\mu$ unnecessarily complicated. One could eliminate this multiple t-dependence in Eq. (24) by making use of Eq. (11); however, this would be a lengthy and awkward procedure. Instead we derive an alternative form of the πNN equations which uses a "free" Green function which contains no two-body interactions and which leads to simpler expressions for the πNN electromagnetic transition currents.

The πNN equations Eqs. (11), (12) and (13), can be written in the form

$$\begin{pmatrix} T_{NN} & \bar T_N \\ T_N & T \end{pmatrix} = \begin{pmatrix} V_{NN} & \bar{\mathcal{F}} \\ \mathcal{F} & L G_0^{-1} \end{pmatrix} \left[1 + \begin{pmatrix} \frac{1}{2} D_0 & 0 \\ 0 & G_0 t G_0 \end{pmatrix} \begin{pmatrix} T_{NN} & \bar T_N \\ T_N & T \end{pmatrix} \right]. \tag{33}$$

where $V_{NN} = V^d_{NN} A$ is given by Eq. (14), and

$$\mathcal{F} = \begin{pmatrix} \mathcal{F}^d_1 A \\ \mathcal{F}^d_3 A \end{pmatrix} = \begin{bmatrix} (F_2 - B^d + F^d_c) A \\ (F_1 + F_2 - B^d + F^d_c) A \end{bmatrix} = \begin{bmatrix} F_2 A - B^{LR} + F_c \\ (F_1 + F_2) A - B^{LR} + F_c \end{bmatrix}, \tag{34}$$

$$\begin{aligned} \bar{\mathcal{F}} &= \left(A \bar{\mathcal{F}}^d_1 \;\; A \bar{\mathcal{F}}^d_3 \right) = \left[A(\bar F_2 - \bar B^d + \bar F^d_c) \;\;\; A(\bar F_1 + \bar F_2 - \bar B^d + \bar F^d_c) \right] \\ &= \left[(A \bar F_2 - \bar B^{LR} + \bar F_c) \;\;\; (A \bar F_1 + A \bar F_2 - \bar B^{LR} + \bar F_c) \right], \end{aligned} \tag{35}$$

$$L = \begin{pmatrix} -P & A \\ A & 0 \end{pmatrix}, \qquad t = \begin{pmatrix} t_1 d_2^{-1} & 0 \\ 0 & \frac{1}{4} t_3 d_3^{-1} \end{pmatrix}. \tag{36}$$

With the view of gauging Green function versions of πNN transition amplitudes, we introduce the Green function matrix appropriate to Eq. (33):

$$\begin{aligned} \mathcal{G} &= \begin{pmatrix} G_{NN} & \bar{G}_N \\ G_N & G \end{pmatrix} \equiv \begin{pmatrix} G_{NN} & G_{N\Delta} & G_{Nd} \\ G_{\Delta N} & G_{\Delta\Delta} & G_{\Delta d} \\ G_{dN} & G_{d\Delta} & G_{dd} \end{pmatrix} \\ &= \begin{pmatrix} AD_0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} D_0 & 0 \\ 0 & G_0 \end{pmatrix} \begin{pmatrix} T_{NN} & \bar{T}_N \\ T_N & T \end{pmatrix} \begin{pmatrix} D_0 & 0 \\ 0 & G_0 \end{pmatrix}. \end{aligned} \tag{37}$$

The inhomogeneous term is chosen so that G_{NN} corresponds exactly to the full Green function for NN scattering. Then, as shown in the Appendix, $\mathcal{G}$ satisfies the equation

$$\mathcal{G} = \begin{pmatrix} AD_0 & 0 \\ 0 & LG_0 \end{pmatrix} \left[1 + \begin{pmatrix} \frac{1}{4} V_{NN} - \frac{1}{4} \bar{\Lambda} L G_0 \Lambda & \frac{1}{2}\bar{\Lambda} \\ \frac{1}{2}\Lambda & t \end{pmatrix} \mathcal{G} \right] \tag{38}$$

where Λ and $\bar{\Lambda}$ are defined by Eqs. (75). The essential feature of Eq. (38) is that it is written in terms of an effective "free" Green function matrix

$$\mathcal{G}_0 = \begin{pmatrix} AD_0 & 0 \\ 0 & LG_0 \end{pmatrix} \tag{39}$$

which does not involve two-body interactions. For this reason Eq. (38) is ideal for the purposes of gauging.

C. Gauging the alternative form of the πNN equations

In terms of the elements of $\mathcal{G}$, the Green function versions of the physical amplitudes [defined in Eq. (26)] are given by

$$\tilde{X}_{NN} = G_{NN} \ ; \quad \tilde{X}_{dN} = \bar{\phi}_d G_{dN} \ ; \quad \tilde{X}_{Nd} = G_{Nd} \phi_d \ ; \quad \tilde{X}_{dd} = \bar{\phi}_d G_{dd} \phi_d. \tag{40}$$

After gauging, these equations give

$$\tilde{X}^\mu_{NN} = G^\mu_{NN} \tag{41}$$

$$\tilde{X}^\mu_{dN} = \bar{\phi}^\mu_d G_{dN} + \bar{\phi}_d G^\mu_{dN} \tag{42}$$

$$\tilde{X}^\mu_{Nd} = G^\mu_{Nd} \phi_d + G_{Nd} \phi^\mu_d \tag{43}$$

$$\tilde{X}^\mu_{dd} = \bar{\phi}^\mu_d G_{dd} \phi_d + \bar{\phi}_d G^\mu_{dd} \phi_d + \bar{\phi}_d G_{dd} \phi^\mu_d. \tag{44}$$

The πNN electromagnetic transition currents $j^\mu_{\alpha\beta}$ are then determined by Eqs. (27) and Eqs. (28).

To determine the quantities $G^\mu_{\alpha\beta}$ in Eqs. (41)-(44) we need to derive the expression for $\mathcal{G}^\mu$ by gauging Eq. (38). Defining

$$\mathcal{V}_t = \begin{pmatrix} \frac{1}{4} V_{NN} - \frac{1}{4} \bar{\Lambda} L G_0 \Lambda & \frac{1}{2}\bar{\Lambda} \\ \frac{1}{2}\Lambda & t \end{pmatrix} \tag{45}$$

Eq. (38) can be written as

$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0 \mathcal{V}_t \mathcal{G}. \tag{46}$$

Gauging this equation and solving for $\mathcal{G}^\mu$ gives

$$\mathcal{G}^\mu = (1 + \mathcal{G}\mathcal{V}_t)\mathcal{G}_0^\mu (1 + \mathcal{V}_t \mathcal{G}) + \mathcal{G}\mathcal{V}_t^\mu \mathcal{G}. \tag{47}$$

To simplify this equation we cannot use Eq. (46) to write $1 + \mathcal{V}_t\mathcal{G} = \mathcal{G}_0^{-1}\mathcal{G}$ and $1 + \mathcal{G}\mathcal{V}_t = \mathcal{G}\mathcal{G}_0^{-1}$ since L is singular so that the inverse $\mathcal{G}_0^{-1}$ does not exist. Instead we use the fact that $L = L\Omega L$, where

$$\Omega = \begin{pmatrix} -\frac{1+P}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{4} \end{pmatrix} \tag{48}$$

which allows us to write

$$\mathcal{G}_0^\mu = \mathcal{G}_0 \mathcal{M}^\mu \mathcal{G}_0 \tag{49}$$

where

$$\mathcal{M}^\mu = \begin{pmatrix} \frac{1}{2} D_0^{-1} D_0^\mu D_0^{-1} & 0 \\ 0 & \Omega G_0^{-1} G_0^\mu G_0^{-1} \end{pmatrix}. \tag{50}$$

Using Eq. (49) in Eq. (47) allows us to write a compact expression for $\mathcal{G}^\mu$:

$$\mathcal{G}^\mu = \mathcal{G}\left(\mathcal{M}^\mu + \mathcal{V}_t^\mu\right)\mathcal{G}. \tag{51}$$

Comparing this expression with the one of Eq. (24), we see that both involve the gauged two-body t matrix t^μ; however, in contrast to Eq. (24), the above expression does not contain adjoining strong interaction t matrices and may therefore be preferable for numerical calculations.

As $D_0^\mu = (d_1 d_2)^\mu = d_1^\mu d_2 + d_1 d_2^\mu$ and $G_0^\mu = (d_1 d_2 d_3)^\mu = d_1^\mu d_2 d_3 + d_1 d_2^\mu d_3 + d_1 d_2 d_3^\mu$, we obtain

$$D_0^{-1} D_0^\mu D_0^{-1} = \Gamma_1^\mu d_2^{-1} + d_1^{-1}\Gamma_2^\mu \tag{52}$$
$$G_0^{-1} G_0^\mu G_0^{-1} = \Gamma_1^\mu d_2^{-1} d_3^{-1} + d_1^{-1}\Gamma_2^\mu d_3^{-1} + d_1^{-1} d_2^{-1}\Gamma_3^\mu \tag{53}$$

where

$$\Gamma_i^\mu = d_i^{-1} d_i^\mu d_i^{-1} \tag{54}$$

is the electromagnetic vertex function of particle i. The term $\mathcal{M}^\mu$ of Eq. (51) thus corresponds to photon coupling in the impulse approximation. The gauged matrix $\mathcal{V}_t^\mu$ corresponds to the interaction currents and consists of the elements $V^\mu = (V_{NN} - \bar{\Lambda} L G_0 \Lambda)^\mu$, $\bar{\Lambda}^\mu$, Λ^μ and t^μ. It is important to note that the diagonal elements of matrix t^μ are both of the form

$$\left(t_i d_j^{-1}\right)^\mu = t_i^\mu d_j^{-1} + t_i \left(d_j^{-1}\right)^\mu = t_i^\mu d_j^{-1} - t_i \Gamma_j^\mu \tag{55}$$

where the last equality follows from the fact that $\left(d_j^{-1} d_j\right)^\mu = 0$. Thus the diagonal elements of t^μ involve new subtraction terms $t_i \Gamma_j^\mu$ whose origin does not lie in the subtraction terms of

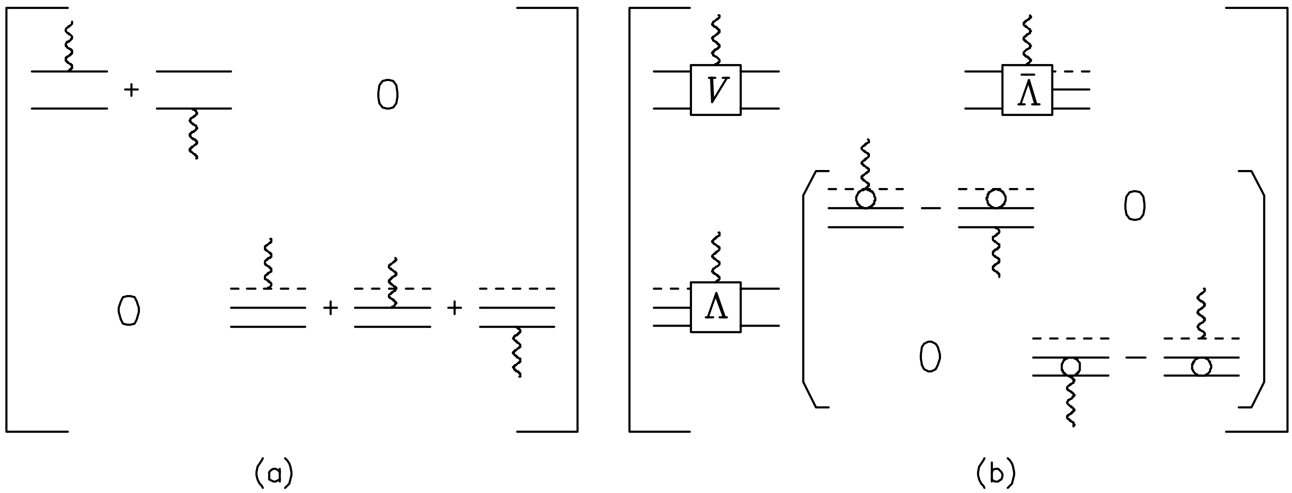

FIG. 6. Graphical representation of (a) $\mathcal{M}^\mu$ and (b) $\mathcal{V}_t^\mu$ which enter into the expression for the gauged Green function matrix $\mathcal{G}^\mu$ of Eq. (51). Constant factors and effects of operator A and matrix Ω have been ignored in this illustration.

the strong interaction πNN equations, but rather in the gauging procedure itself. Analogous subtraction terms arise in the three-nucleon problem whose strong interaction equations have no subtraction terms [1,2]. Similar subtraction terms will arise in the gauging of F_1 and $\bar{F}_1$ contained in Λ and $\bar{\Lambda}$ respectively. The graphical representation of $\mathcal{M}^\mu$ and $\mathcal{V}_t^\mu$ is given in Fig. 6.

Eq. (51) can be used to determine all the possible electromagnetic transition current of the πNN system. For example, current j^μ_{NN} for the electromagnetic transition $NN \rightarrow NN$ is given by

$$
\begin{aligned}
j^\mu_{NN} &= D_0^{-1} G^\mu_{NN} D_0^{-1} && (56)\\
&= \sum_{\alpha\beta} D_0^{-1} G_{N\alpha} \left(\mathcal{M}^\mu + \mathcal{V}_t^\mu\right)_{\alpha\beta} G_{\beta N} D_0^{-1} \\
&= \sum_{\alpha\beta} \left(A\delta_{N\alpha} + T_{N\alpha} D_\alpha\right) \left(\mathcal{M}^\mu + \mathcal{V}_t^\mu\right)_{\alpha\beta} \left(A\delta_{\beta N} + D_\beta T_{\beta N}\right) && (57)
\end{aligned}
$$

where $\alpha, \beta = N, \Delta, d$, and $D_N \equiv D_0$, $D_\Delta = D_d \equiv G_0$.

An especially interesting use of Eq. (51) is to study the electromagnetic properties of bound states of the πNN system. It is certainly expected that the strong interaction πNN model under discussion admits a bound state corresponding to the physical deuteron. In this case a solution will exist to the homogeneous version of Eq. (11):

$$
\Phi = \mathcal{V} \mathcal{G}_t \Phi \tag{58}
$$

where Φ is a matrix of deuteron vertex functions

$$
\Phi = \begin{pmatrix} \Phi_N \\ \Phi_\Delta \\ \Phi_d \end{pmatrix}. \tag{59}
$$

Here Φ_N is the usual deuteron vertex function describing the $d \rightarrow NN$ transition, while Φ_Δ and Φ_d are somewhat unusual in that they describe transitions to clustered πNN states: $d \rightarrow (\pi N)N$

and $d \to (NN)\pi$ respectively. Comparing Eq. (37) and Eq. (38) it is seen that $\mathcal{G}$ has a pole at the deuteron mass M_d:

$$\mathcal{G} \sim i\frac{\Psi\bar{\Psi}}{P^2 - M_d^2} \qquad \text{as} \quad P^2 \to M_d^2 \tag{60}$$

where P is the total four-momentum of the system, and where Ψ satisfies the equation

$$\Psi = \mathcal{G}_0\mathcal{V}_t\Psi. \tag{61}$$

Clearly Ψ is related to Φ by the equation

$$\Psi = \begin{pmatrix} D_0 & 0 \\ 0 & G_0 \end{pmatrix} \Phi \tag{62}$$

so that either of the equations (58) or (61) can be used to determine Ψ. Taking the left and right residues at the deuteron bound state poles of Eq. (51) we obtain the bound state electromagnetic current

$$j^\mu = \bar{\Psi}\left(\mathcal{M}^\mu + \mathcal{V}_t^\mu\right)\Psi \tag{63}$$

which describes the electromagnetic properties of the deuteron whose internal structure is described by the present πNN model. Eq. (63) provides a very rich description of the internal electromagnetic structure of the deuteron with all possible meson exchange currents being taken into account in a gauge invariant way. In view of the accuracy of this model which is based on meson and baryon degrees of freedom, a comparison of the deuteron electromagnetic form factors (easily extracted from j^μ) with those extracted from experiment should prove to be most interesting.

IV. CONCLUSIONS

We have derived gauge invariant expressions for the electromagnetic transition currents of the πNN system where the strong interactions are described by four-dimensional integral equations. The feature of our approach is that the external photon is coupled everywhere in the strong interaction model, in this way giving a unified description of the πNN and $\gamma\pi NN$ systems. This unified approach to the πNN system and its electromagnetic currents has been made possible by the recent introduction of the gauging of equations method [1,2]. The use of this method has also enabled us to avoid all the overcounting problems that are inherent in four-dimensional descriptions of πNN-like systems [6].

The expressions we have derived can be used directly to make four-dimensional calculations of all the reactions induced in the πNN system by an external electromagnetic probe. However, in view of the practical difficulty of solving four-dimensional integral equations, it may also be useful to have a gauge invariant three-dimensional description of the same processes. In that case our four-dimensional expressions can be used to provide the starting point for a three-dimensional reduction. One such three-dimensional reduction scheme that preserves gauge invariance and is easily applied to our four-dimensional expressions was discussed in Ref. [3].

Although we have specifically gauged the πNN system, it should be noted that the derived expressions apply equally well to other systems consisting of two fermions and one boson which

can be absorbed by the fermions. For example one could apply our expressions to the quark-antiquark-gluon system in order to calculate the meson spectrum including its electromagnetic properties.

It is also worth noting that our derivation of the gauged πNN equations only assumed that the external field couples to hadrons only to first order in the field-hadron coupling constant, but otherwise does not depend on the nature of the external field. Thus, for example, our expressions can be used directly to determine the weak interaction transition currents of the πNN system.

ACKNOWLEDGEMENTS

The authors would like to thank the Australian Research Council for their financial support.

APPENDIX

In this Appendix we derive Eq. (38). Writing Eq. (33) as $\mathcal{T} = \mathcal{V} + \mathcal{V}\mathcal{G}_t\mathcal{T}$ and Eq. (37) as $\mathcal{G} = \mathcal{G}_u + g\mathcal{T}g$ where

$$\mathcal{G}_u = \begin{pmatrix} AD_0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad g = \begin{pmatrix} D_0 & 0 \\ 0 & G_0 \end{pmatrix}, \tag{64}$$

$$\mathcal{V} = \begin{pmatrix} V_{NN} & \bar{\mathcal{F}} \\ \mathcal{F} & LG_0^{-1} \end{pmatrix}, \qquad \mathcal{G}_t = \begin{pmatrix} \frac{1}{2}D_0 & 0 \\ 0 & G_0tG_0 \end{pmatrix}, \tag{65}$$

we have that

$$\mathcal{G} = \mathcal{G}_u + g\mathcal{T}g = \mathcal{G}_u + g\mathcal{V}g + g\mathcal{V}\mathcal{G}_t\mathcal{T}g = \mathcal{G}_u + g\mathcal{V}g + g\mathcal{V}\mathcal{G}_tg^{-1}(\mathcal{G} - \mathcal{G}_u) \tag{66}$$

or

$$\mathcal{G} = \mathcal{G}_u + g\mathcal{V}\left(g - \mathcal{G}_tg^{-1}\mathcal{G}_u\right) + g\mathcal{V}\mathcal{G}_tg^{-1}\mathcal{G}. \tag{67}$$

Using the explicit matrix forms for $\mathcal{G}_u$, g, $\mathcal{V}$, and $\mathcal{G}_t$ given in Eq. (64) and Eq. (65), the equation for $\mathcal{G}$ takes the form

$$\mathcal{G} = \begin{pmatrix} AD_0 & D_0\bar{\mathcal{F}}G_0 \\ 0 & LG_0 \end{pmatrix} + \begin{pmatrix} \frac{1}{2}D_0V_{NN} & D_0\bar{\mathcal{F}}G_0t \\ \frac{1}{2}G_0\mathcal{F} & LG_0t \end{pmatrix}\mathcal{G}. \tag{68}$$

Multiplying this equation from the left by the matrix

$$R = \begin{pmatrix} 1 & -D_0\bar{\mathcal{F}}Z \\ 0 & ZL \end{pmatrix} \tag{69}$$

where

$$Z = \begin{pmatrix} -\frac{1+P}{2} & \frac{A}{4} \\ \frac{A}{4} & -\frac{A}{8} \end{pmatrix} \qquad \text{so that} \qquad ZL = LZ = \begin{pmatrix} 1 & 0 \\ 0 & \frac{A}{2} \end{pmatrix}, \tag{70}$$

we obtain

$$\begin{pmatrix} 1 & -D_0\bar{\mathcal{F}}Z \\ 0 & ZL \end{pmatrix} \mathcal{G} = \begin{pmatrix} AD_0 & 0 \\ 0 & LG_0 \end{pmatrix} + \begin{pmatrix} D_0 & 0 \\ 0 & LG_0 \end{pmatrix} \begin{pmatrix} \frac{1}{2}V_{NN} - \frac{1}{2}\bar{\mathcal{F}}G_0 Z\mathcal{F} & 0 \\ \frac{1}{2}Z\mathcal{F} & t \end{pmatrix} \mathcal{G}. \qquad (71)$$

Transferring the off-diagonal term $-D_0\bar{\mathcal{F}}Z$ from the left hand side to the right and recognising that

$$\begin{pmatrix} 1 & 0 \\ 0 & LZ \end{pmatrix} \mathcal{G} = \mathcal{G} \qquad (72)$$

Eq. (71) becomes

$$\mathcal{G} = \begin{pmatrix} AD_0 & 0 \\ 0 & LG_0 \end{pmatrix} + \begin{pmatrix} D_0 & 0 \\ 0 & LG_0 \end{pmatrix} \begin{pmatrix} \frac{1}{2}V_{NN} - \frac{1}{2}\bar{\mathcal{F}}G_0 Z\mathcal{F} & \bar{\mathcal{F}}Z \\ \frac{1}{2}Z\mathcal{F} & t \end{pmatrix} \mathcal{G}. \qquad (73)$$

This equation may be written in the form

$$\mathcal{G} = \begin{pmatrix} AD_0 & 0 \\ 0 & LG_0 \end{pmatrix} \left[1 + \begin{pmatrix} \frac{1}{4}V_{NN} - \frac{1}{4}\bar{\Lambda}LG_0\Lambda & \frac{1}{2}\bar{\Lambda} \\ \frac{1}{2}\Lambda & t \end{pmatrix} \mathcal{G} \right]. \qquad (74)$$

where

$$\Lambda = \left\{ \begin{matrix} \left[F_1 - \frac{1}{2}(B^d - F_c^d)\right] A \\ -\frac{1}{2}(B^d - F_c^d) \end{matrix} \right\}; \qquad \bar{\Lambda} = \left\{ A\left[\bar{F}_1 - \frac{1}{2}(\bar{B}^d - \bar{F}_c^d)\right] \quad -\frac{1}{2}(\bar{B}^d - \bar{F}_c^d) \right\}. \qquad (75)$$

We note that the presence of the antisymmetrization operator A in the term AD_0 of Eq. (74) allows one to eliminate the antisymmetrization operators in V_{NN} and $\bar{\Lambda}$ by making the replacements

$$V_{NN} \to 2V_{NN}^d \qquad \text{and} \qquad A\left[\bar{F}_1 - \frac{1}{2}(\bar{B}^d - \bar{F}_c^d)\right] \to 2\left[\bar{F}_1 - \frac{1}{2}(\bar{B}^d - \bar{F}_c^d)\right] \qquad (76)$$

in Eq. (74) and Eq. (75) respectively.

REFERENCES

[1] A. N. Kvinikhidze and B. Blankleider, *Coupling photons to hadronic processes*, invited talk at the Joint Japan Australia Workshop, Quarks, Hadrons and Nuclei, November 15-24, 1995 (unpublished); in Proceedings of the XVth International Conference on Few-Body Problems in Physics, Groningen, The Netherlands, July 22-26, 1997, published in Nucl. Phys. **A631**, 559c (1998).

[2] A.N. Kvinikhidze and B. Blankleider, *Gauging the three-nucleon system*, in preparation.

[3] A.N. Kvinikhidze and B. Blankleider, Phys. Rev. C **56**, 2963 (1997).

[4] A.N. Kvinikhidze and B. Blankleider, Phys. Rev. C **56**, 2973 (1997).

[5] B. Blankleider and A. N. Kvinikhidze, in Proceedings of the APCTP Workshop on Astro-Hadron Physics, "Properties of Hadrons in Matter", Seoul, Korea, 25-31 October, 1997, to be published by World Scientific, nucl-th/9802017.

[6] A.N. Kvinikhidze and B. Blankleider, Nucl. Phys. **A574**, 788 (1994).

[7] D. R. Phillips and I. R. Afnan, Ann. Phys. (N.Y.) **247**, 19 (1996).

[8] J. G. Taylor, Nuovo Cimento Suppl. **1**, 857 (1963); Phys. Rev. **150**, 1321 (1966).

[9] A. Tucciarone, Nuovo Cimento **41**, 204 (1966).

[10] M. M. Broido, Rep. Prog. Phys. **32**, 493 (1969).

[11] Y. Avishai and T. Mizutani, Phys. Rev. C **27**, 312 (1983).

[12] I. R. Afnan and B. Blankleider, PRC **32**, 2006 (1985).

[13] H. Haberzettl, Phys. Rev. C **47**, 1237 (1993); *ibid.* **49**, 2142 (1994).

[14] D. R. Phillips and I. R. Afnan, Ann. Phys. (N.Y.) **240**, 266 (1995).

[15] I. R. Afnan and B. Blankleider, Phys. Rev. C **22**, 1638 (1980).